$\frac{17}{120}+\frac{7}{256}=\frac{649}{3840}$, C has $\frac{11}{420}$, D has $\frac{11}{28}-\frac{7}{128}=\frac{303}{896}$, and E has $\frac{157}{1680}$; eight, A has $\frac{51}{1280}-\frac{21}{2048}+\frac{1}{15}=\frac{2957}{30720}$, B has $\frac{649}{3840}-\frac{21}{2048}+\frac{1}{15}=\frac{6925}{30720}$, C has $\frac{11}{420}+\frac{7}{1024}+\frac{1}{15}=\frac{10719}{107520}$, D has $\frac{303}{896}+\frac{7}{1024}+\frac{1}{15}=\frac{44263}{107520}$, and E has $\frac{157}{1680}+\frac{7}{1024}+\frac{1}{15}=\frac{17951}{107520}$, or reducing these fractions to a common denominator, we have the following: $A\ \frac{20699}{215040}$, $B\ \frac{48475}{215040}$, $C\ \frac{21438}{215040}$, $D\ \frac{88526}{215040}$, $E\ \frac{35902}{215040}$, the sum of which is $\frac{215040}{215040}=1$ as it should be.

Excellent solutions of this problem were received from *G. B. M. Zerr*, *E. W. Morrell*, and *P. S. Berg*

ERRATUM—In the solution of problem 42, Professor Cooper D. Schmitt's address should read, Professor of Mathematics, University of Tenn. etc.

PROBLEMS.

48. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Fifty thousand days preceding Thursday, March 7, 1895, was what date and what day of the week?

49. Proposed by J. A. CALDERHEAD, B. Sc., Superintendent of Schools, Limaville, Ohio.

I have a garden in the form of an equilateral triangle, whose sides are 200 feet. At each corner stands a tower; the height of the first is 30 feet, the second is 40 feet, and the third is 50 feet. At what distance from the base of each tower must a ladder be placed, that it may just reach the top of each? And what is the length of the ladder, the garden being a horizontal plane?

[From *Greenleaf's National Arithmetic.*]

Give a solution simple enough to be presented to a class in arithmetic.

ALGEBRA.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

39. Proposed by ARTEMAS MARTIN, LL. D., U. S. Coast aud Geodetic Survey Office, Washington, D. C.

Find x, y, z, and w from the equations

$$x^4+y^4+z^4+x^2+y^2+z^2=112\ldots.(1),$$
$$x^4+z^4+w^4+x^2+z^2+w^2=382\ldots.(2),$$
$$x^4+y^4+w^4+x^2+y^2+w^2=294\ldots.(3),$$
$$y^4+z^4+w^4+y^2+z^2+w^2=364\ldots.(4).$$

I. Solution by A. H. BELL, Hillsboro, Illinois, P. S. BERG, Apple Creek, Ohio, D. G. DURRANCE, Jr., Camden. N.Y., COOPER D. SCHMITT, A.M., University of Tennessee, and H.C.WILKES, Murraysville, West Virginia.

Adding the four equations and dividing the result by 3, we readily obtain, $x^2+y^2+z^2+w^2+x^4+y^4+z^4+w^4=384$. From this subtract each equation in order and we obtain $w^2+w^4=272$, $y^2+y^4=2$, $z^2+z^4=90$, $x^2+x^4=20$, all of which are bi-quadratics. Solving we find, $w=\pm4$ or $\pm\sqrt{-17}$, $y=\pm1$ or $\pm\sqrt{-2}$, $z=\pm3$ or $\pm\sqrt{-10}$, $x=\pm2$ or $\pm\sqrt{-5}$.

II. **Solution by LEONARD E. DICKSON, M. A., University of Chicago.**

$$\begin{array}{lll}
(2)-(1) \text{ gives } w^4+w^2-y^4-y^2=270 & & (5)\\
(3)-(1) \text{ gives } w^4+w^2-z^4-z^2=182 & & (6)\\
(4)-(1) \text{ gives } w^4+w^2-x^4-x^2=252 & & (7)\\
(5)-(6) \text{ gives } z^4+z^2-y^4-y^2=\ 88 & & (8)\\
(5)-(7) \text{ gives } x^4+x^2-y^4-y^2=\ 18 & & (9)\\
(1)-(9) \text{ gives } 2y^4+2y^2+z^4+z^2=94 & & (10)\\
(10)-(8) \text{ gives } 3y^4+3y^2=6 & & (11)\\
\therefore\ y=\pm1 \text{ or } \pm\sqrt{-2}. & &
\end{array}$$

From (8) and (11), $z^4+z^2=90$. $\therefore z=\pm3$ or $\pm\sqrt{-10}$.
From (9) and (11), $x^4+x^2=20$. $\therefore x=\pm2$ or $\pm\sqrt{-5}$.
From (5) and (11), $w^4+w^2=272$. $\therefore w=\pm4$ or $\pm\sqrt{-17}$.
Hence there are $4^4=256$ sets of values as solutions.

Also solved by *B. F. Burleson, H. W. Draughon, J. H. Drummond, J. K. Ellwood, M. A. Gruber, J. F. W. Scheffer, F. P. Matz*, and *G. B. M. Zerr*.

40. **Proposed by B. F. BURLESON, Oneida Castle, New York.**

Find by quadratics all the possible values for x and y in the equations $x^3+y^3=b=35, \ldots.(1)$, and $x^2+y^2=a=13\ldots.(2)$.

I. Solution by the PROPOSER.

From equation (1) $y=\sqrt[3]{(b-x^3)}\ldots.(3)$. From equation (2) $y=\sqrt{(a-x^2)}\ldots.(4)$. Equating (3) and (4) and clearing from radicals we obtain, $2x^6-3ax^4-2bx^3+3a^2x^2-(a^3-b^2)=0\ldots.(5)$. Substituting numerical for literal values in (5), it becomes, $2x^6-39x^4-70x^3+507x^2-972=0\ldots.(6)$. Factoring (6), $(x^2-5x+6)(x^2-2x-4\frac{1}{2})(2x^2+14x+36)=0\ldots.(7)$. Thus far finding the six roots of equation (6), it is resolved into finding the roots of the three quadratic equations $x^2-5x=-6\ldots.(8)$, $x^2-2x=4\frac{1}{2}\ldots.(9)$, and $2x^2+14x=-36\ldots.(10)$. Resolving equation (8) for the two values of x in it, and then substituting these values severally in (3) or (4) for the corresponding values of y, we get, $x=2$ or 3, and $y=3$ or 2. In the same way we find from eq. (9), $x=1+\sqrt{5\frac{1}{2}}=3.345208+$ and $y=1-\sqrt{5\frac{1}{2}}=-1.345208+$, or $x=1-\sqrt{5\frac{1}{2}}=-1.345208+$, and $y=1+\sqrt{5\frac{1}{2}}=3.345208+$. From eq. (10) we obtain the imaginary roots, $x=-3\frac{1}{2}+\frac{1}{2}\sqrt{-23}$, and $y=-3\frac{1}{2}-\frac{1}{2}\sqrt{-23}$, $x=-3\frac{1}{2}-\frac{1}{2}\sqrt{-23}$ and $y=-3\frac{1}{2}+\frac{1}{2}\sqrt{-23}$. Thus x and y have six values each and no more, all of which we have found by quadratics.

II. Solution by J. K. ELLWOOD, A. M., Colfax-School, Pittsburg, Pennsylvania, and J. W. WATSON, Middle Creek, Ohio.

Let $x+y=p$, $xy=s$. Then the equations become, $ap-sp=b$, $a+2s=p^2$. Eliminating s, $p^3-3ap+2b=0$, or $p^3-39p+70=0$. It seems the literal solution can not be completed by using quadratics. But multiplying p^3-39p